$$\left(\frac{\partial\theta}{\partial x}\right)_0 = -\sqrt{\frac{2}{n_t+2}m_p^2\,\theta_p^2\left[\left(\frac{\theta_0}{\theta_p}\right)^{n_t+2}-1\right]+\left(\frac{\partial\theta}{\partial x}\right)_p^2} \quad (6b)$$

$$\left(\frac{\partial\theta}{\partial x}\right)_p \cong -\sqrt{\frac{2}{n+2}m_i^2\,\theta_i^2\left[\left(\frac{\theta_p}{\theta_i}\right)^{n+2}-1\right]} \quad (6c)$$

Equation (6*b*) is similar to Equation (6) with the exception that n, θ_i, and m_i are replaced by n_t, θ_p and m_p, respectively. Equation (6*c*) is an approximation similar to Equation (6*a*) wherein θ_p and $(\partial\theta/\partial x)_p$ replace θ_0 and $(\partial\theta/\partial x)_0$.

The parameter values for isopropanol and freon are given in Table 1. The calculated results from Equations (6*b*) and (6*c*) are shown in Figures 5 and 6. Agreement with experimental data is greatly improved.

Thus the simplified method proposed in this report can be used to estimate the temperature gradient at the fin base to the first-order approximation, or it can be used to provide the initial values for a more elaborate computer program such as that used in reference *1*. If moderate accuracy is adequate, then Equation (6*a*) may be used, which still is vastly more simple than the exact numerical solution.

CONCLUSION

A simple model is proposed to determine the length of the nucleate boiling section on a fin and the base heat flux of the fin as a function of the ratios of heat transfer coefficients of nucleate boiling and convective modes, as well as the characteristics of the fin. Qualitative agreement is observed between the analytical and the experimental results.

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NOTATION

A = $2\sigma T_{sat}/\lambda\rho_v$
b = half-width of fin
C = coefficient
C_3 = constant, 1.6
h = heat transfer coefficient
K = thermal conductivity
L = length of fin
L_i = length of boiling section
M = parameter, Equation (11)
m = $\sqrt{\bar{h}_b/Kb}$
m_i = $\sqrt{h_i/Kb}$
n = exponent used in Equation (2*b*)
q = heat flux
r = radius
T = temperature
T_{bulk} = bulk temperature
x = coordinate shown in Figure 1
y = coordinate shown in Figure 1

Greek Letters

β = L_i/L
γ = $\sqrt{\bar{h}_c/\bar{h}_b}$
δ = limiting thermal layer thickness
θ = $T - T_{bulk}$
λ = latent heat of evaporation
μ = mL
ρ_v = density of saturation vapor
σ = surface tension

Subscripts

b = boiling
c = convection
i = incipience of boiling or transition between convection and nucleate boiling
0 = base
p = transition from nucleate to transition boiling
sat = saturation
t = transition boiling region

Superscript

— = average value

LITERATURE CITED

1. Haley, K. W., and J. W. Westwater, *Proc. Third Intern. Heat Transfer Conf.*, AIChE-ASME, 3, 245 (1966).
2. Cumo, M., S. Lopez, and G. C. Pinchera, *Chem. Eng. Progr. Symp. Ser. No.* 59, **61**, 225 (1965).
3. Hsu, Y. Y., *J. Heat Transfer*, **84**, 207 (1962).
4. Dunskus, T., and J. W. Westwater, *Chem. Eng. Progr. Symp. Ser. No.* 32, **57**, 173 (1961).
5. Haley, K. W., and J. W. Westwater, *Chem. Eng. Sci.*, **20**, 711 (1965).

Free Oscillations of Fluids in Manometers

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Biery (*1, 2*) recently presented the results of a numerical and experimental study of free oscillations of fluids in U shaped manometer tubes. His work included the use of Newtonian and non-Newtonian fluids. In order to obtain good agreement between calculations and experiments, Biery found it necessary to introduce a correction for end effects which was based upon physical concepts but which could not be evaluated on its own; the heuristic correction effectively may be only a fitting factor, and comparisons with other experiments and analyses provide an important check upon the results. It is shown here that the experimental results of Biery do not differ significantly from those of previous investigations, and it is suggested that the correction factor requires reexamination.

The study of oscillations in U tubes has been curiously beset by misunderstandings and neglect of previous work. The first extensive investigation by Menneret (3) occurred shortly after the fundamental paper in boundary-layer theory, which it could have utilized; however, even in the late 1940's the boundary-layer character of some U tube oscillations was not well appreciated. Von Karman and Valensi contributed to elucidation of this point (4). The analysis of the flow for small oscillation Reynolds numbers ($\omega_p r_o^2/\nu$) was made by several investigators (5). Ury repeated analysis of the problem in terms of a Bessel function series expansion (6). A discussion of the flow regimes is found (7), and further observations of vortices are reported by Binnie (8). Biery, in turn, has not used any of this previous work.

The oscillation Reynolds number can be interpreted as proportional to the square of the ratio of the tube radius to the oscillation boundary-layer thickness, $(\nu/\omega_p)^{1/2}$. Thus, for large Reynolds numbers the viscous effects are concentrated near the walls, and for small Reynolds numbers the boundary layer has become so large relative to the radius that the velocity gradients are nowhere small. The amplitude decay can be determined as a function of the oscillation Reynolds number, provided that effects of surface tension and tube curvature can be neglected. For the boundary-layer case ($N_{Re} > 70$, say) the logarithmic decrement is given by (4)

$$\frac{\lambda}{\omega_p} = \frac{1}{\sqrt{2}}\left[\frac{r_o^2\omega_p}{\nu}\right]^{-1/2} \tag{1}$$

and for the tube-filling case (5)

$$\frac{\lambda}{\omega_p} = 2.892\left[\frac{r_o^2\omega_p}{\nu}\right]^{-1} \tag{2}$$

Correlation of Menneret's results for the latter case gave a coefficient of 3.95 instead of 2.892. Biery's experimental results agree with those of Menneret very well.

THE CHOICE OF THE DECREMENT FACTOR

An ambiguity arises in making comparisons between expressions involving logarithmic decrements as a function of $(r_o^2\omega_p/\nu)$ and experiments or analysis where the decrement is expressed as a function of $(r_o^2\omega/\nu)$. The equation for one-dimensional simple harmonic motion with linear velocity damping is

$$\frac{d^2h^*}{dt^{*2}} + 2\omega_n\zeta\frac{dh^*}{dt^*} + \omega_n^2 h^* = 0 \tag{3}$$

where ω_n is the natural frequency in the completely undamped state ($\zeta = 0$). The logarithmic decrement is $(\zeta\pi/q)$, where $q = \omega_n(1-\zeta^2)^{1/2}$; that is, the logarithmic decrement is $\zeta\pi$ divided by the (circular) frequency in the damped state. The shift of the cycle frequency down from ω_p to q as a result of damping is appropriate to a simple, one-dimensional situation. However, in the case of the U tube fluid oscillations, the cycle frequency is also changed because of the development of the transverse velocity distribution. The undamped cycle frequency ω_s calculated from $\omega_s = \omega_o(1-\zeta^2)^{-1/2}$ (where ω_o is the observed or computed frequency), is not equal to ω_p. It is therefore ambiguous how data obtained from experiments or computations should be expressed for plotting and comparison with the asymptotic boundary-layer analysis, for example.

It seems pertinent to ask what the physical significance of ζ is for the manometer. If Equation (3) is thought of as an overall momentum balance equation for the liquid column, the coefficient $(2\omega_n\zeta)$ represents the viscous shear along the column divided by the column mass. However, this shear cannot be identified simply with the wall shear, since this is out of phase with the oscillations except as $(r_o^2\omega_p/\nu) \to 0$; this shear factor must be determined in terms of an integral across the tube, so that the velocity distribution would have to be known to permit interpretation of ζ in terms of the viscosity. Thus it does not seem more useful to express the damping in terms of ζ rather than the logarithmic decrement Δ directly:

$$\Delta = \frac{1}{\pi}\ln\left|\frac{x_m}{x_{m+1}}\right|$$

where x_m, x_{m+1} are the extremal amplitudes of successive half-cycles. For nonlinear damping, the effect of friction on the equivalent one-dimensional frequency is not allowed for correctly by the relation

$$\zeta^2/(1-\zeta^2) = \Delta^2$$

BIERY'S COMPUTED RESULTS WITH K_T OMITTED

It is of interest to compare the decrements determined by the computer program, without use of the K_T empirical correction, and Equations (1) and (2). At small N_{Re}, the computed decrements exceeded those of Equation (2) but were below the correlation for Menneret. At large N_{Re}, the computed decrements showed a trend towards Equation (1) from above, but the computed decrements did not fit completely smoothly to a single curve. This is due partly to the fact that some of the computed results are associated with the first half of the first cycle of oscillation, while the remainder are for the second half of the first cycle. Each of these two halves has different initial conditions for the velocity field, so that small differences in the decrement can be expected. Table 1 illustrates this effect for the five largest values of $(r_o^2\omega_p/\nu)$: $(\zeta - \lambda/\omega_p)$ is roughly twice as large, in proportion to $(\nu/\omega r_o^2)$, for runs 20 and 21 as for the other runs listed. It is possible that the forward-difference equation used was not computed in such a way as to give uniformly valid results. One potentially unsatisfactory feature of the computations in (1) is that for large N_{Re}, the damping factors were estimated with use of a one-third factor rule based on computations with grid size, in time and space, changed by a factor of 2; unfortunately the rule cannot be justified as a general characteristic. At the largest Reynolds number involved the grid size ($\Delta r^* = 0.0125$), quoted as giving results which still fell short of the final extrapolated value, is of the order of one-tenth of

TABLE 1.

Run No.	$r_o^2\omega_p/\nu$	Logarithmic decrement λ/ω_p von Karman and Valensi	ζ, calculated Biery without K_T	$\lambda/\omega_p - \zeta$
20	779.5	0.02533	0.0262	0.00087
31	571.4	0.02958	0.0305	0.00092
21	167.2	0.05468	0.0613	0.00762
32	122.6	0.06387	0.0729	0.00903
41	107.6	0.06817	0.0793	0.01113

the oscillation boundary-layer thickness, and an interval as coarse as this may well be inadequate to represent the flow structure. Computations directed at boundary-layer problems commonly involve intervals of 1/100 to 1/1,000 of the boundary-layer thickness.

ARE THE VALUES OF K_T REASONABLE?

It is important to assess the reasonableness of the values of K_T. The correction factor K_T is introduced by Biery (*1*) as a factor in a term [his Equation (18)] to be subtracted from the right-hand sides of his Equations (13) and (14). The latter two equations are finite-difference expressions, and this makes it difficult to try to assess the reasonableness of Biery's values for K_T directly. However, it is possible to assess this in another way. If the equation for damped oscillations, Equation (3), is considered, it is obvious that the principal contribution to the damping factor ζ is the viscous force per unit length of tube times the liquid column length. Any increase in ζ could be interpreted as an increase in the effective viscous length of the column, other things being held constant. A reasonable value for the equivalent added length to account for flow reversal at the end is of the order of the tube radius. However, examination of Tables 3 and 4 in (*1*) shows that the correction length applied is of the order of ten times the radius. Thus the identification of the differences between experimental and numerical results as being due to flow reversal effects needs better justification before it can be accepted.

Another difficulty arises when the end correction is considered at large values of $(r_o^2\omega_p/\nu)$. For large values, the momentum in column-end vortices is dissipated slowly, so that they will continue spinning in the same direction for a while after the column direction of motion has changed (that is, the vortex spin becomes shifted in phase from the main motion). This will lead to different end effects for the first and second half-cycles: the first half-cycle does not inherit any end vortices. This effect does not seem to have been accounted for by Biery in his use of K_T. It is not clear from Biery's paper (*1*) whether the experimental ζ used in determining the correction factors K_T were obtained at the same stage in the oscillation sequence as the corresponding calculations, that is, the first or second half of the first cycle (see Table 4 of reference *1*). It may be noted that the largest absolute differences in logarithmic decrement occur at the smallest Reynolds numbers, while the vortex motion with which K_T has been associated has been observed at moderate and large Reynolds numbers (*7*). It is important that the accounting of corrections be based firmly upon solutions of the equations of fluid motion, for the specific forms of motion involved, if this type of viscometer is to be used as a primary instrument. This point appears particularly important in this case, since we do not have a sound basis for assuming that secondary motions which occur in Newtonian fluids and may cause corrections to the amplitude decay will occur with the same relative strength in non-Newtonian fluids under roughly comparable conditions. Indeed, we may expect to find in general that secondary motions are significantly different. In consequence of this, the use of the same computational scheme and the same magnitudes of K_T for interpretation of measurements with non-Newtonian fluids is also laid in doubt.

SECONDARY MOTION IN OSCILLATING FLOWS AT LARGE $(r_o^2\omega_p/\nu)$

Biery (*1, 2*) discounted the significance of secondary motion in the curved portion of the U tube on the basis of its magnitude in steady flow. However, the secondary motion in oscillatory flow does not correspond simply to that in steady flow, particularly when the oscillation Reynolds number is large. The phenomenon of acoustic streaming is well known and can be strong enough to account for significant heat transfer, for example. As discussed in Rosenhead (*9*), the characteristic of the oscillating flow needed for the generation of secondary motions is the existence of Reynolds' stresses, and these should be found in the curved portion of a U tube containing an oscillating flow.

SUMMARY

In summary, Biery's measurements with Newtonian fluids do agree, but his corresponding computations do not agree as well as might be hoped, with previous results for the same configuration, and the rationale for empirical correction factor K_T which he determined is put in doubt, as is the subsequent use of the computation program and K_T in interpretation of measurements with non-Newtonian fluids. A satisfactory accounting for the differences between fairly simple analyses and experiment through a fluid-mechanically acceptable solution of the equations of motion remains to be made. If these refinements are carried through, it is not unreasonable to look forward to accuracies of the order of 1% instead of the order of 10%. In more refined oscillation viscometers of the oscillating disk type, accuracies of 0.1% can sometimes be achieved.

The amplitude decay characteristics of oscillating power law fluids may well lead to use of techniques of interpretation appropriate when damping is proportional to some power of the velocity, reviewed recently (*10*). However, in some circumstances it may be difficult to distinguish between the form of transition at large displacement Reynolds number (already observed with Newtonian fluids) and changes in the Ostwald-de Waele index with changes in maximum shear rate.

NOTATION

K_T = an arbitrary constant; reference *1*, Equation (17)
N_{Re} = oscillation Reynolds number, $r_o^2\omega_p/\nu$
r = radius
r^* = r/r_o
r_o = tube radius

Greek Letters

Δ = logarithmic decrement
λ = logarithmic decrement (*4, 5*)
μ = dynamical viscosity
ν = kinematic viscosity, μ/ρ
ρ = fluid density
ω_p = natural circular frequency of oscillation, $(2g/l)^{1/2}$

LITERATURE CITED

1. Biery, J. C., *AIChE J.*, **9**, 606-614 (1963).
2. *Ibid.*, **10**, 551-557 (1964).
3. Menneret, M., Thèse de doctorat des sciences physiques (Paris), Imprimerie Allier Frères, Grenoble (1911).
4. von Karman, T., and J. Valensi, *Compt. Rend. Acad. Sci. Paris*, **227**, 105-106 (1948).
5. Valensi, J., *ibid.*, **224**, 446-448 (1947).
6. Ury, J. F., *Intern. J. Mech. Sci.*, **4**, 349-369 (1962).
7. Valensi, J., C. Clarion, and F. Zerner, *Compt. Rend. Acad. Sci. Paris*, **230**, 2002-2004 (1950).
8. Binnie, A. M., *Proc. Phys. Soc.* (*London*), **57**, 390-402 (1945).
9. Rosenhead, L., ed., "Laminar Boundary Layers," pp. 578-579, Chapt. 7, Oxford Univ. Press, England (1963).
10. Richardson, P. D., *J. Roy. Aeron. Soc.*, **68**, 846-849 (1964).